A Complete and irredundant linear description of the asymmetric traveling salesman polytope on 6 nodes

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A COMPLETE AND IRREDUNDANT LINEAR DESCRIPTION OF THE ASYMMETRIC TRAVELING SALESMAN POLYTOPE ON 6 NODES

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Une description linéaire complète et irredondante du polytope associé au problème du voyageur de commerce asymétrique à 6 sommets

Reinhardt Euler* and Hervé Le Verge

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Résumé : Nous avons obtenu une description complète et irredondante du polytope associé au problème du voyageur de commerce asymétrique à 6 sommets, à l'aide d'une mise-en-œuvre de l'algorithme de Chernikova. Outre les 11 équations définissant l'enveloppe affine de ce polytope, la description obtenue comprend 319015 inégalités.
Mots-clé : optimisation combinatoire, voyageur de commerce asymétrique.

A complete and irredundant linear description of the asymmetric traveling salesman polytope on 6 nodes

Abstract: Using a refined version of Chernikova's algorithm we determined a complete and irredundant description of the asymmetric traveling salesman polytope on 6 nodes. Besides the 11 equations describing the affine hull of this polytope, our description consists of 319015 facet-defining inequalities.
Key-words: combinatorial optimization, asymmetric traveling salesman.

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A complete and irredundant linear description of the asymmetric traveling salesman polytope on 6 nodes

R. Euler* and H. Le Verge†

October 13, 1992

Abstract

Using a refined version of Chernikova's algorithm we determined a complete and irredundant description of the asymmetric traveling salesman polytope on 6 nodes. Besides the 11 equations describing the affine hull of this polytope, our description consists of 319015 facet-defining inequalities.

1 Introduction

The efforts to completely describe small asymmetric traveling salesman polytopes date back to the year 1953 when I. Heller published a (partial) description of this polytope on 5 nodes by means of 224 equations and inequalities [Hel53]. In 1955, H.W. Kuhn [Kuh55] presented two more classes stating that his system of 9 equations and 390 inequalities describes the polytope in an irredundant manner. 34 years later Bartels and Bartels [BB89] confirmed Kuhn's results by use of a computer program.

In this paper, we consider the asymmetric traveling salesman polytope on 6 nodes, which is defined to be the convex hull of the incidence vectors of 120 tours in the complete digraph on 6 nodes. Since we adopt the notation of [Gro77] we denote this polytope by $\boldsymbol{P}_T^6$ and recall that its dimension is 19.

We obtained our results using an implementation of Chernikova's algorithm [Che64], [Che65], [Le 92], which for a given set of extremal rays and vertices determines the associated set of irredundant equations and facet-defining inequalities.

In section 2 we discuss the basic steps of our approach. Section 3 contains the main results. We describe 287 classes of facet-defining inequalities by exhibiting the associated

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'valued graph' (i.e. the directed graph on six nodes containing a distinguished arc for every coefficient that is strictly positive) together with the right-hand side of the inequality. Note that each of the representative inequalities has been verified to be 'support-reduced' (in the sense of Fischetti [Fis91]) and therefore defines a facet of the monotonization of $\boldsymbol{P}_T^6$. We also provide information on:

- the number of different inequalities forming a class (i.e. those which define the same facet of $\boldsymbol{P}_T^6$ up to permutation of the 6 nodes and up to arc-reversal),
- the number of vertices of $\boldsymbol{P}_T^6$ satisfying the inequality with equality, and
- those classes of inequalities that have been described earlier (implicitly or explicitly).

It turns out that the major part (252 classes among 287 altogether) represents facet-defining inequalities that have been unknown to date. We therefore believe that our results will contribute to better describe the general asymmetric traveling salesman polytope (for previous work see [Bal89], [BF90], [Fis91], [Gro77] and [GP85]) and we hope that this will lead to better cutting plane methods for the solution of related applications.

Section 4 contains our conclusions and some comments on related work.

2 The basic steps of our approach

As many other polytopes associated to combinatorial optimization problems the polytope $\boldsymbol{P}_T^6$ is symmetric in the sense that the facets containing one specific vertex can be obtained from the facets containing some arbitrary vertex by a suitable permutation of the 6 nodes. So if we denote by x_1 the incidence vector of the tour $\{(1,2),(2,3),(3,4),(4,5),(5,6),(6,1)\}$ and by $x_2, x_3, \ldots, x_{120}$ those associated with the remaining tours, it is sufficient to consider C, the pointed cone induced by the rays $x_2 - x_1, \ldots, x_{120} - x_1$ and the vertex x_1. To obtain an irredundant description of C by means of linear equations and inequalities Chernikova's algorithm can be applied. It is based on Fourier-Motzkin elimination combined with a particular redundancy check. A description of the implementation of this algorithm can be found in [Le 92]. Note that any ray that is a non-negative linear combination of the remaining ones can be eliminated, and our computational experience showed that it is worthwhile to do so. We used a simplex like procedure to eliminate redundant rays hereby obtaining all those rays $x_i - x_1$, for which x_i is adjacent to x_1 on $\boldsymbol{P}_T^6$.

We thus reduced the original 119 rays to a set of 110 irredundant rays to which we applied our algorithm. As a result we obtained 11 equations and the, remarkable, number of 58799 inequalities fully describing C in an irredundant manner.

The final step consisted in properly identifying all classes of facet-defining inequalities for $\boldsymbol{P}_T^6$. To this end we projected our description of C into a space of dimension 19 which guarantees the uniqueness (up to multiplication by a scalar) of any facet-defining inequality (see [Sch86] for further details). We then considered two such inequalities to belong to a single class whenever one can be obtained from the other by a permutation of the six nodes.

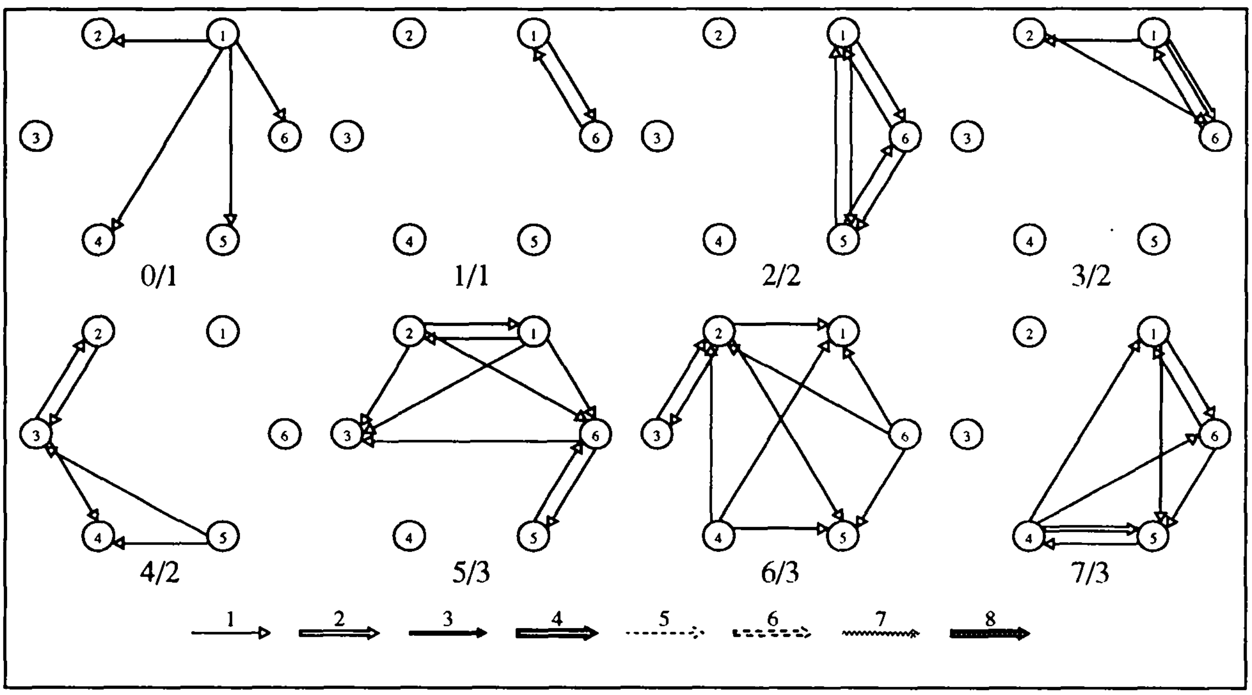
We ended up with 473 classes altogether. To further reduce the number of classes we merged two classes into one whenever one could be obtained from the other by arc-reversal.

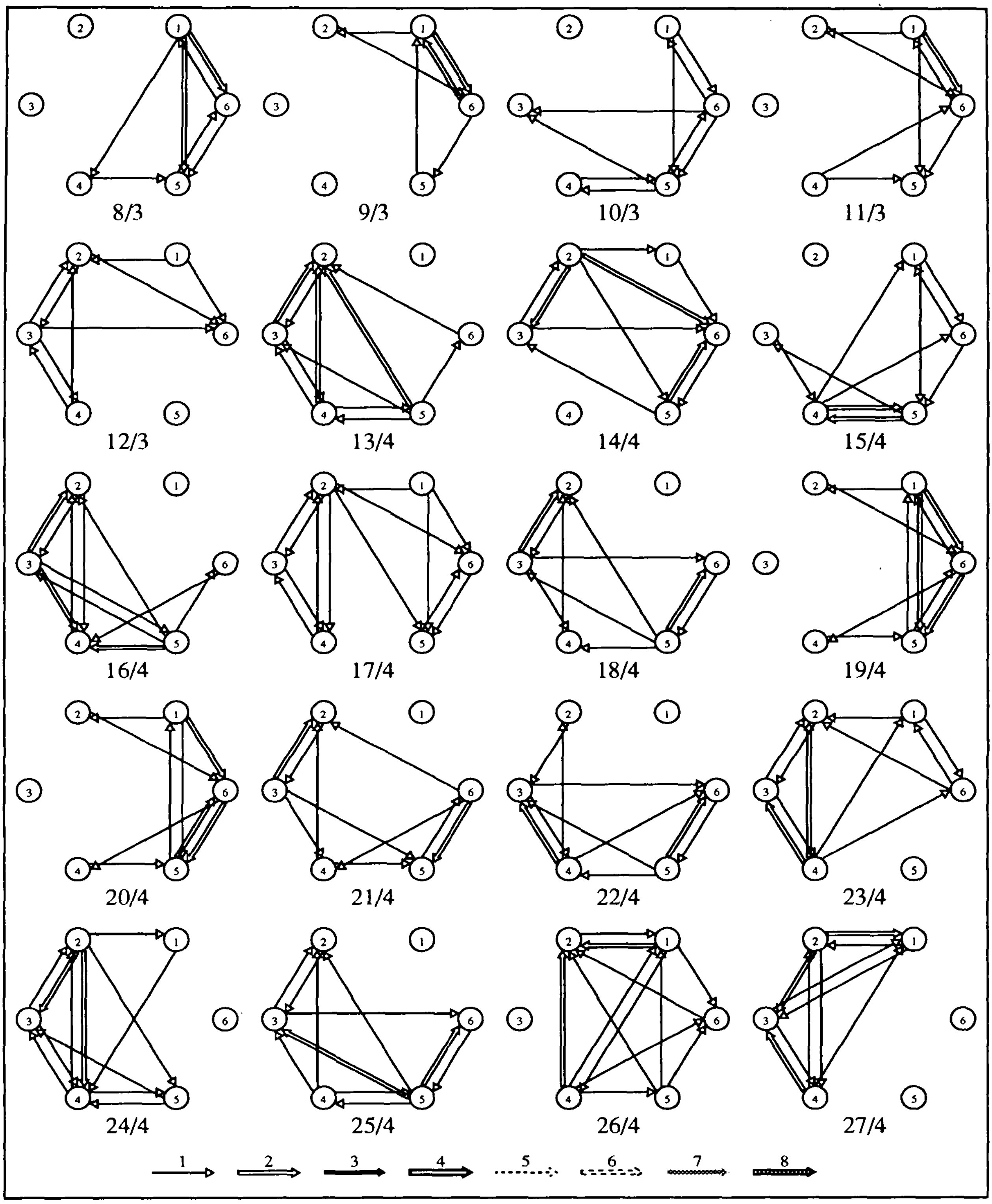
3 The main results

We first exhibit for each of the 287 classes the valued graph associated with one representative facet-defining inequality, that has been chosen according to the following criteria:

- the inequality is support-reduced;
- its right-hand side is minimum;
- if already known, its form corresponds to that used in literature;
- inequalities that, after suppression of one isolated node, are equivalent to one defining a facet of P_T^5, are represented in the corresponding form.

As for the remaining we have chosen a form in which the highest coefficient is minimum. The class number and the right hand-side of the corresponding inequality is indicated under each graph.

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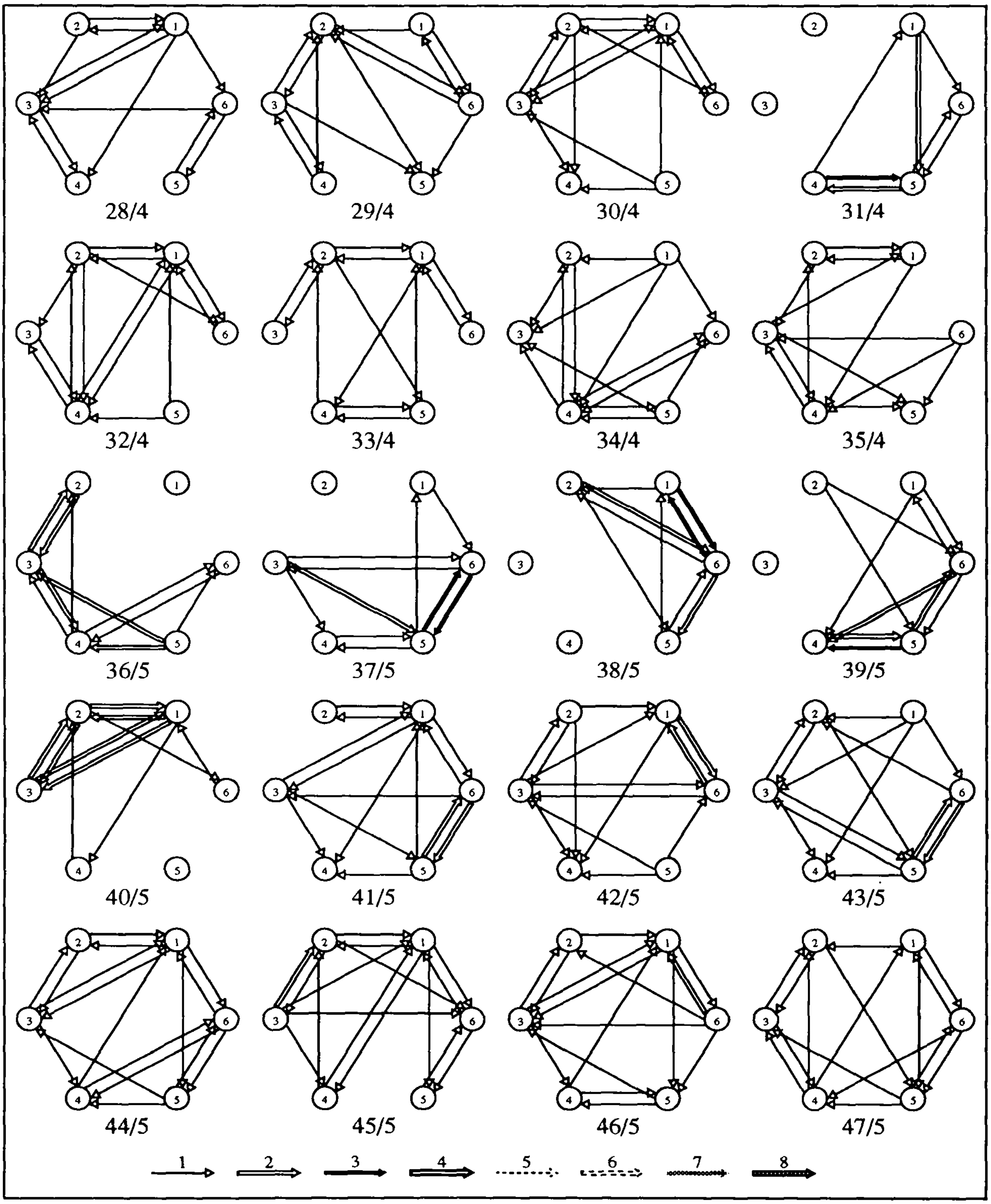
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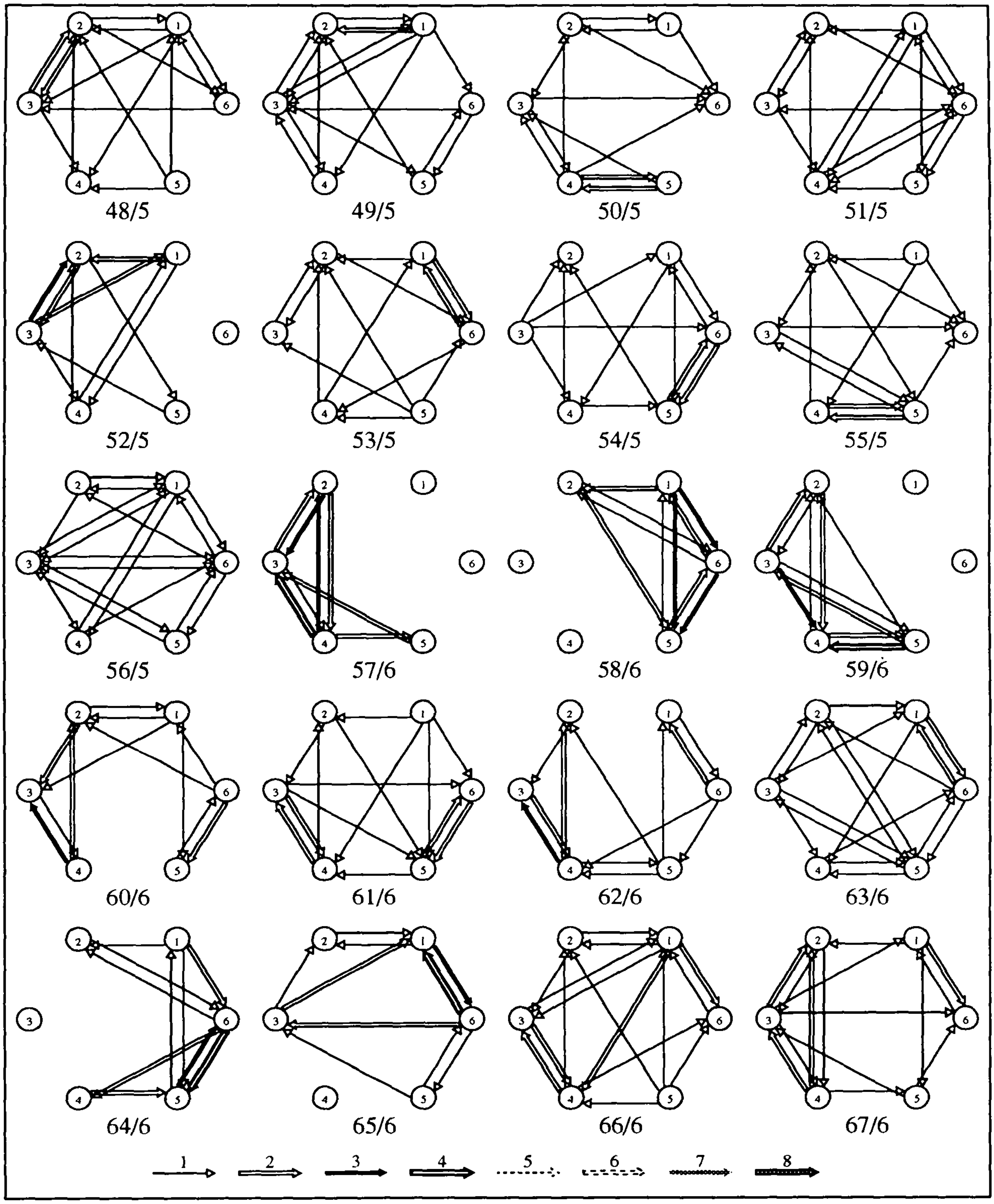
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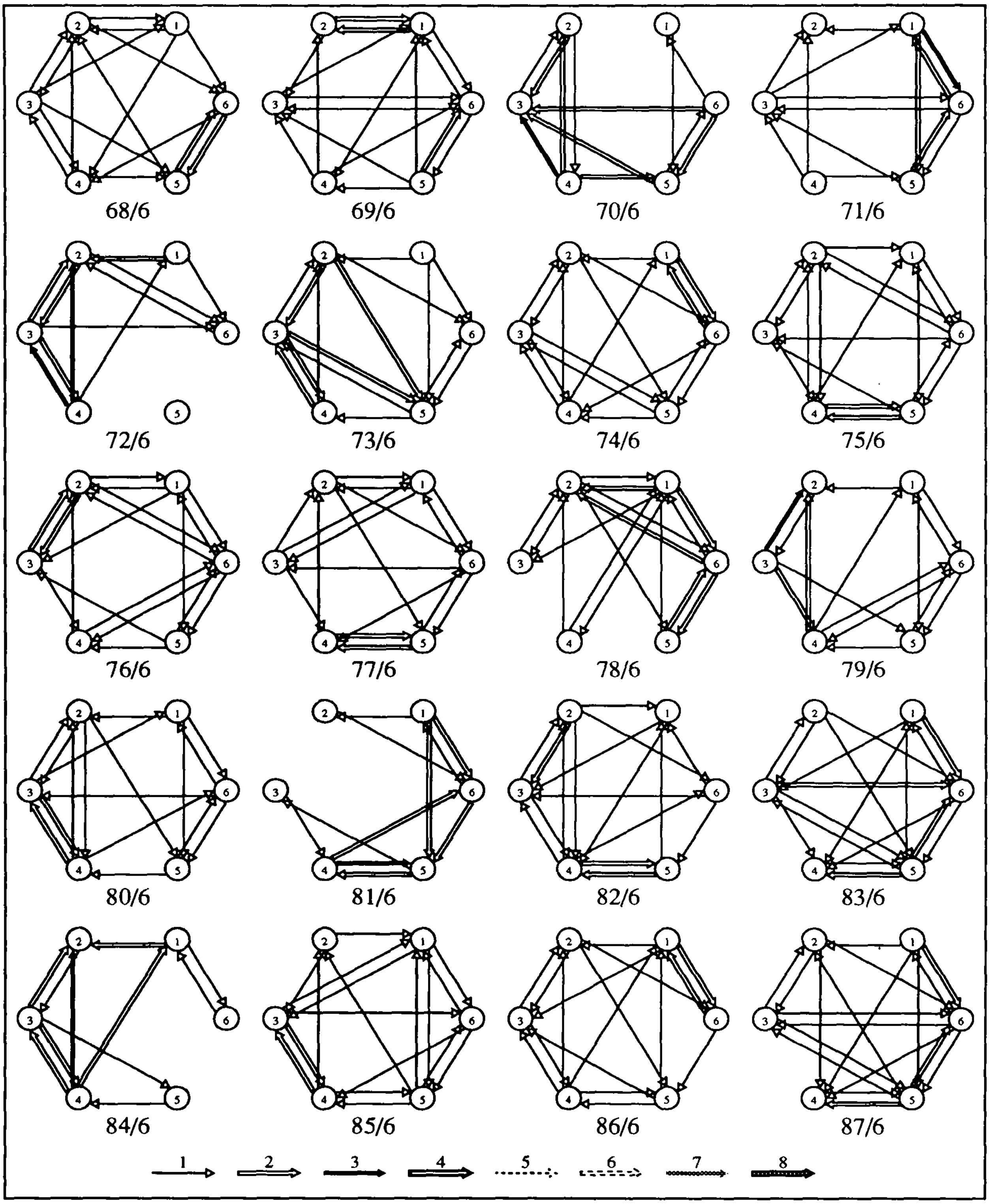
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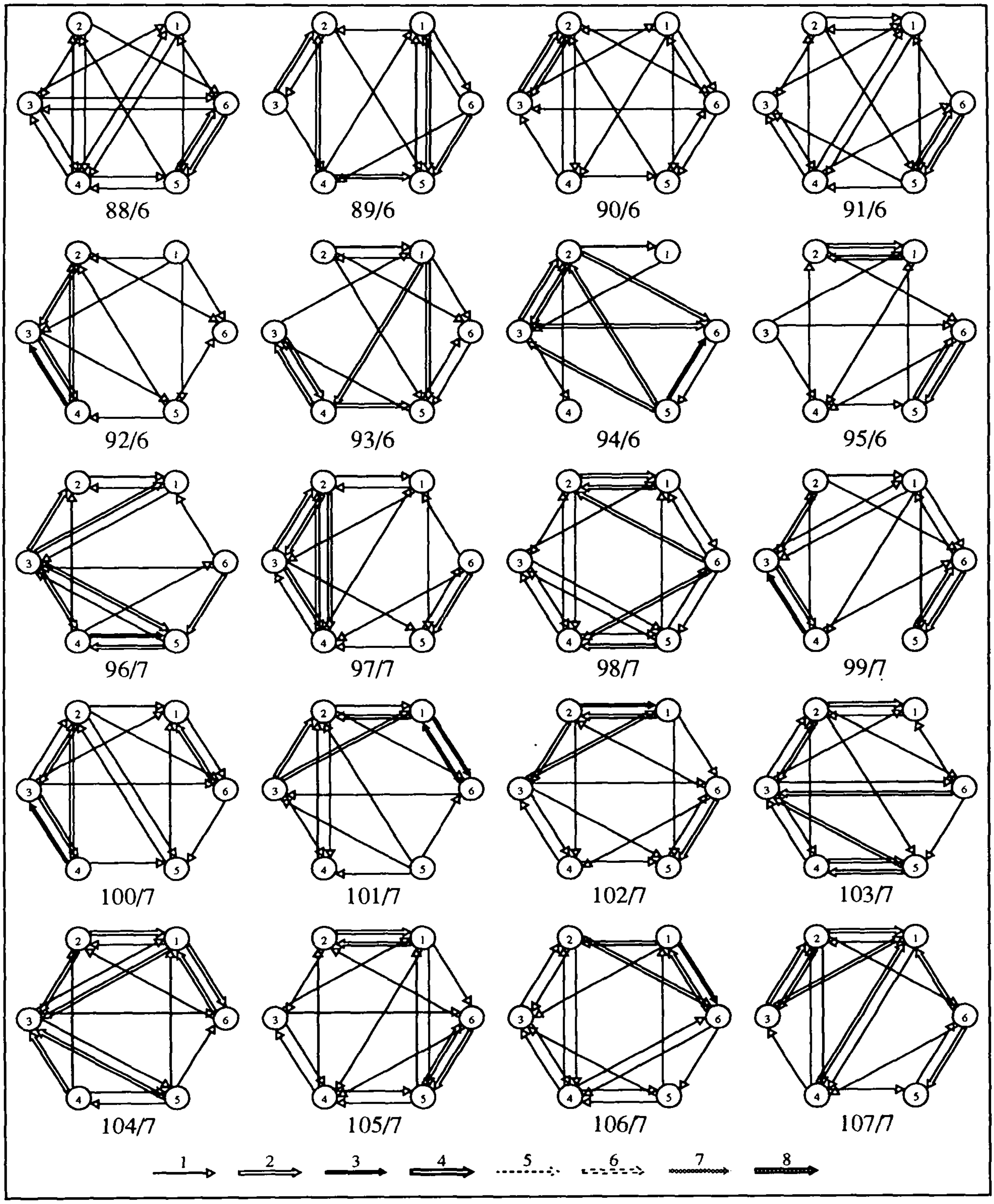
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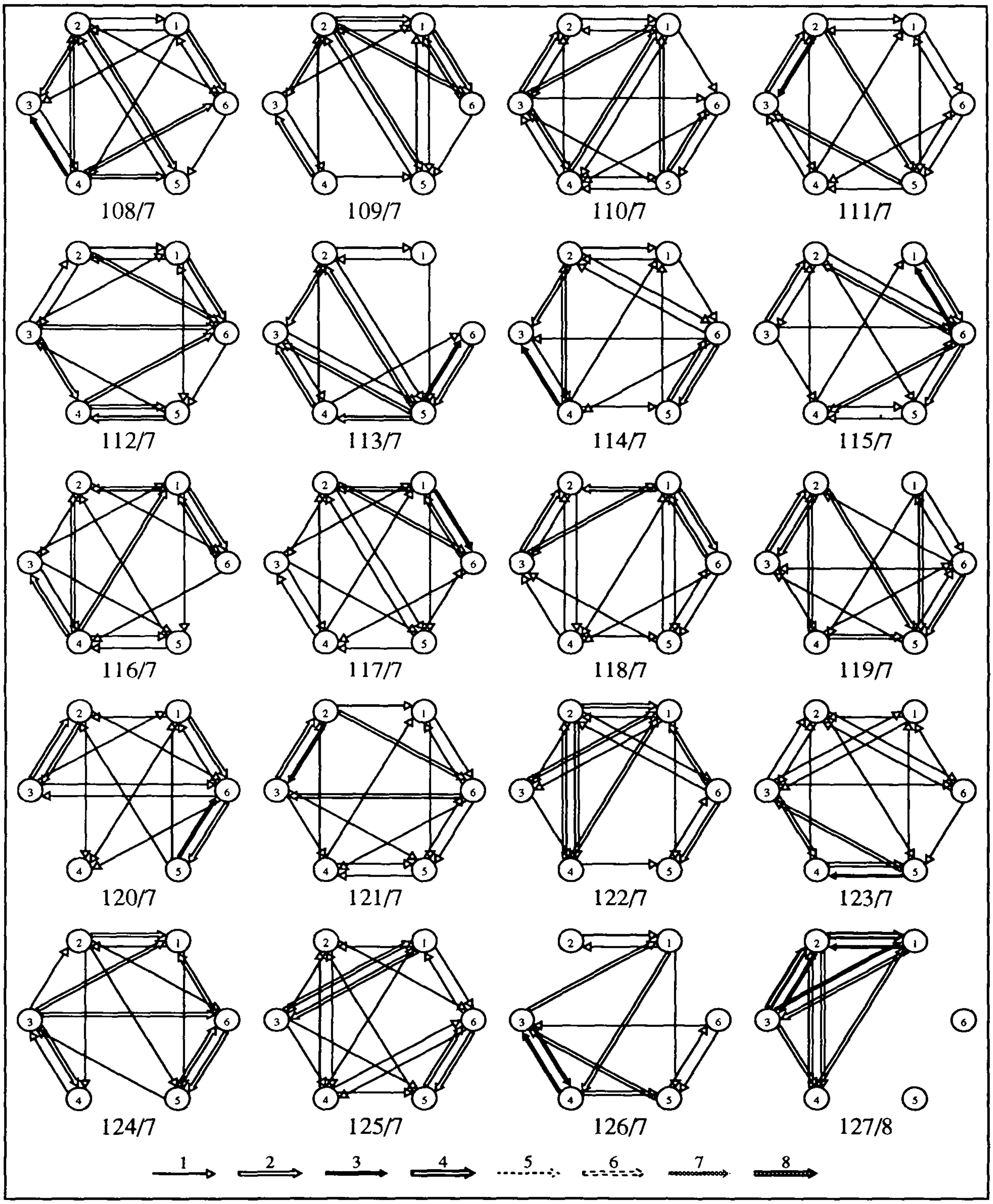
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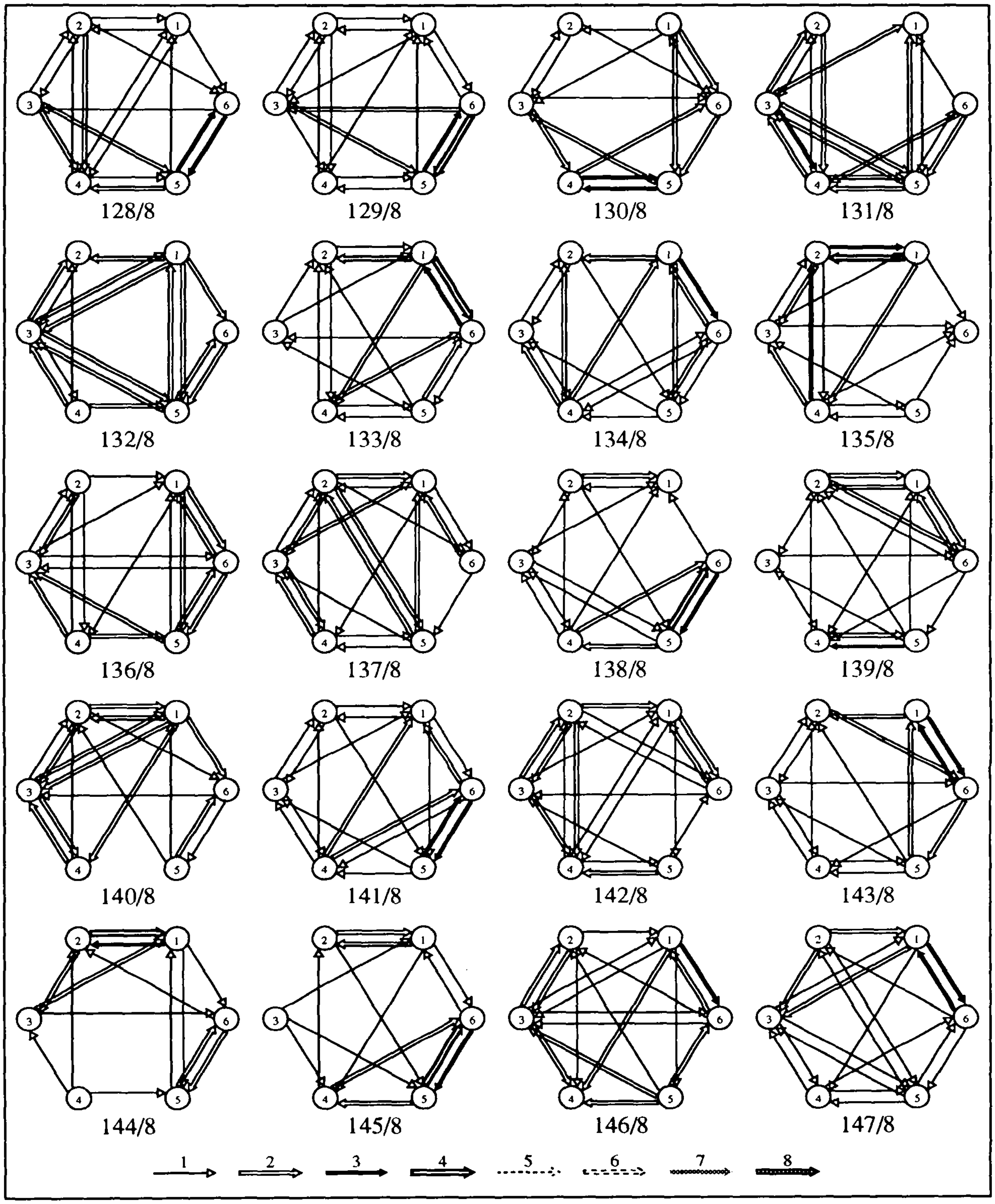
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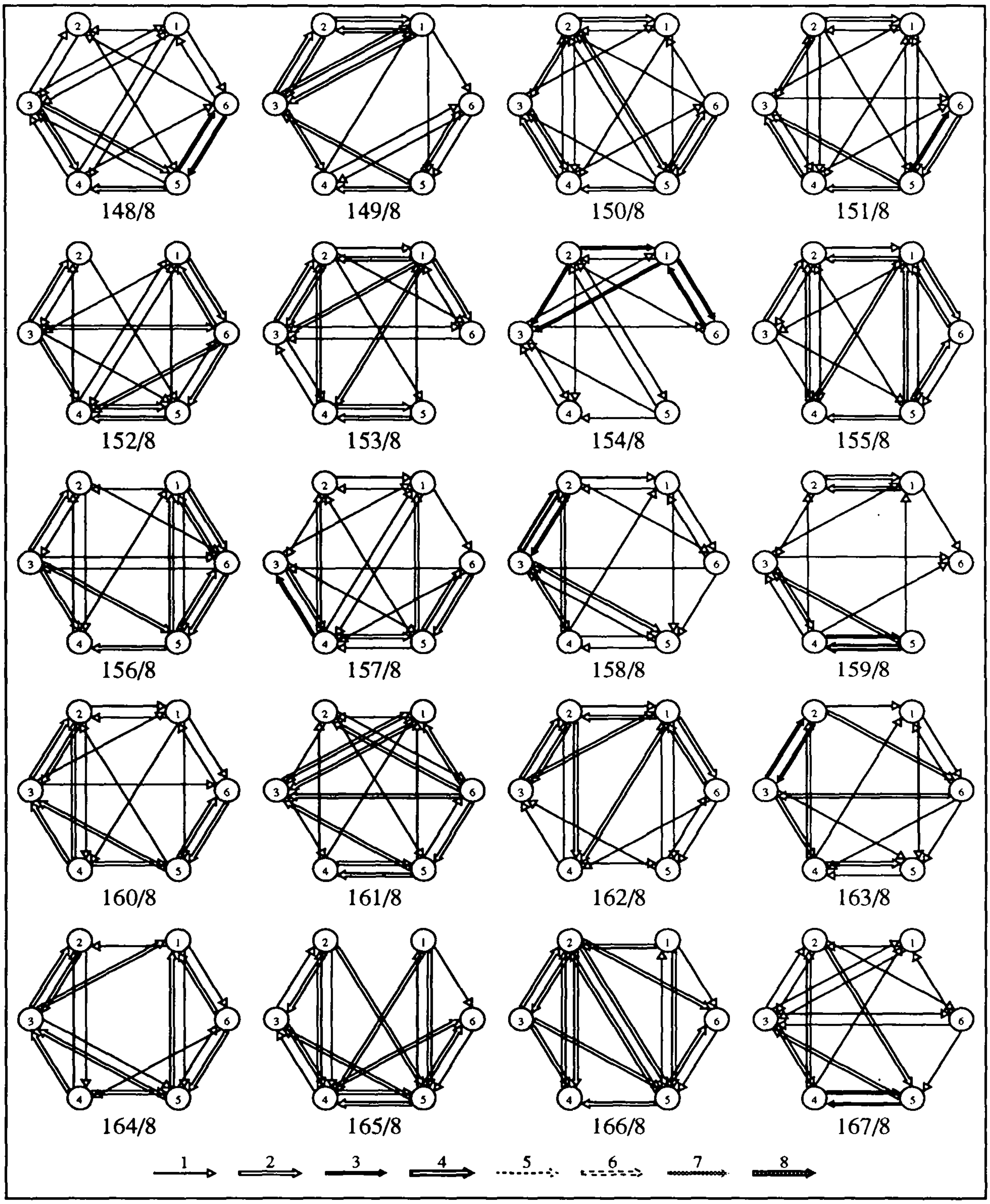
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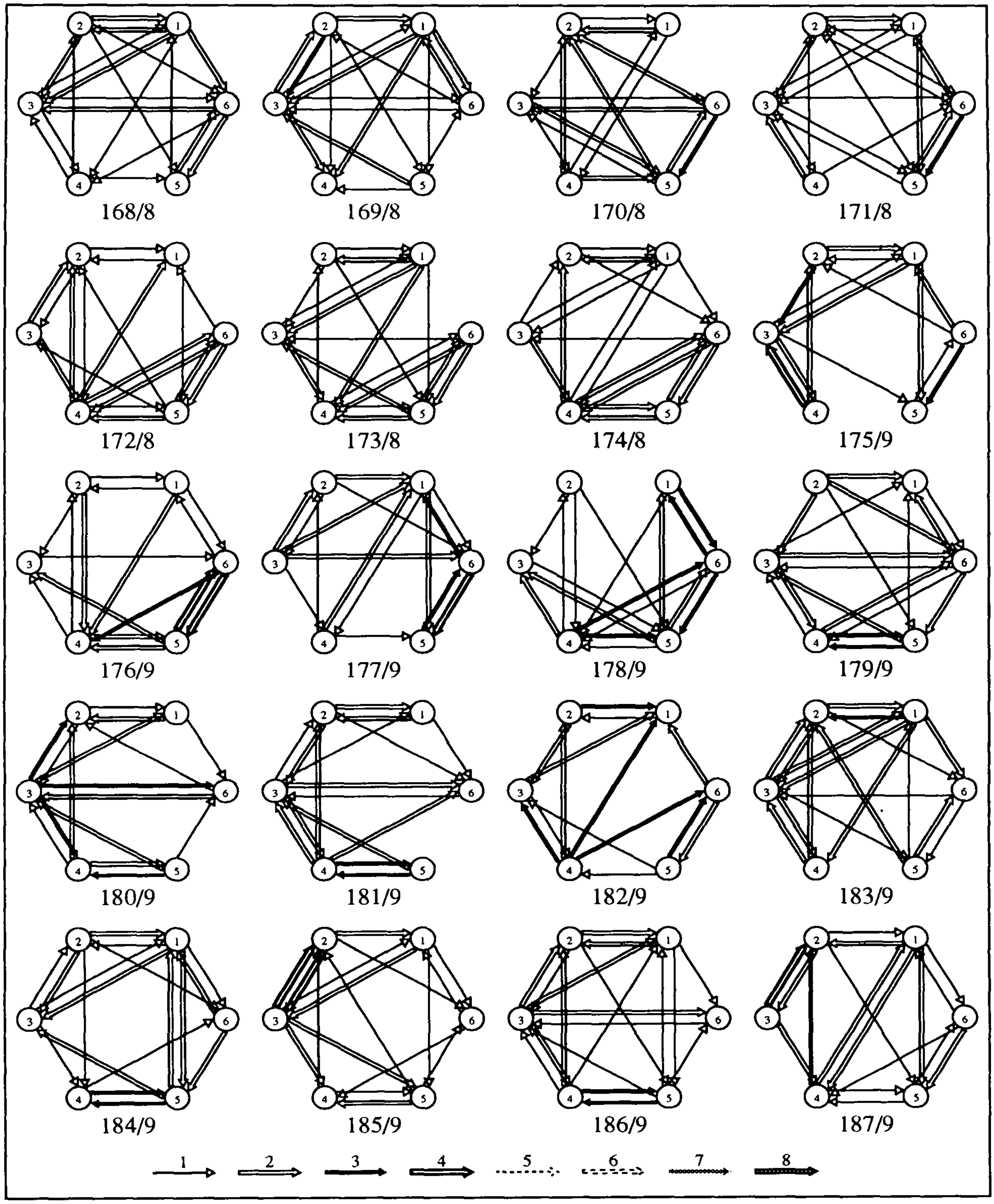
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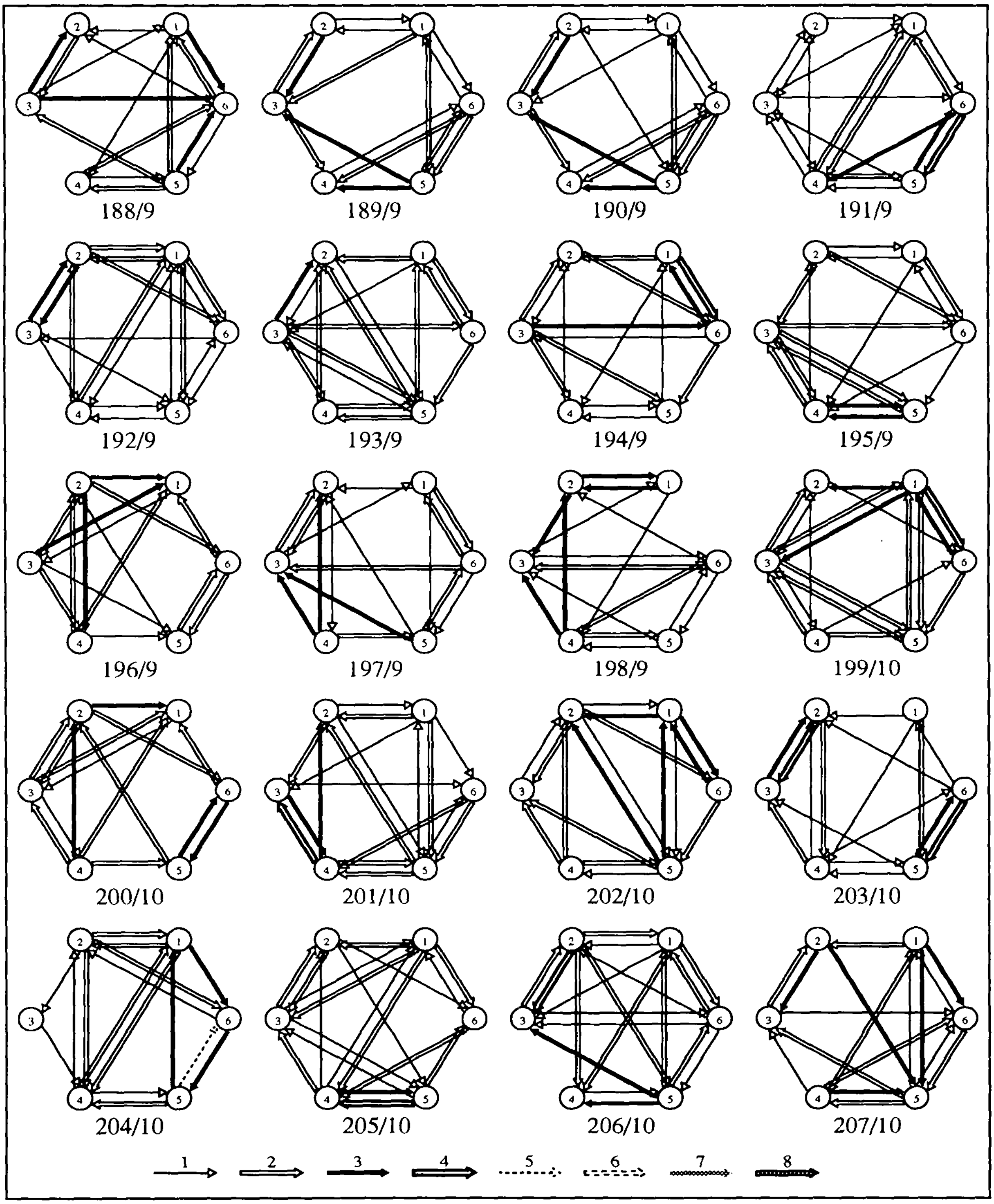

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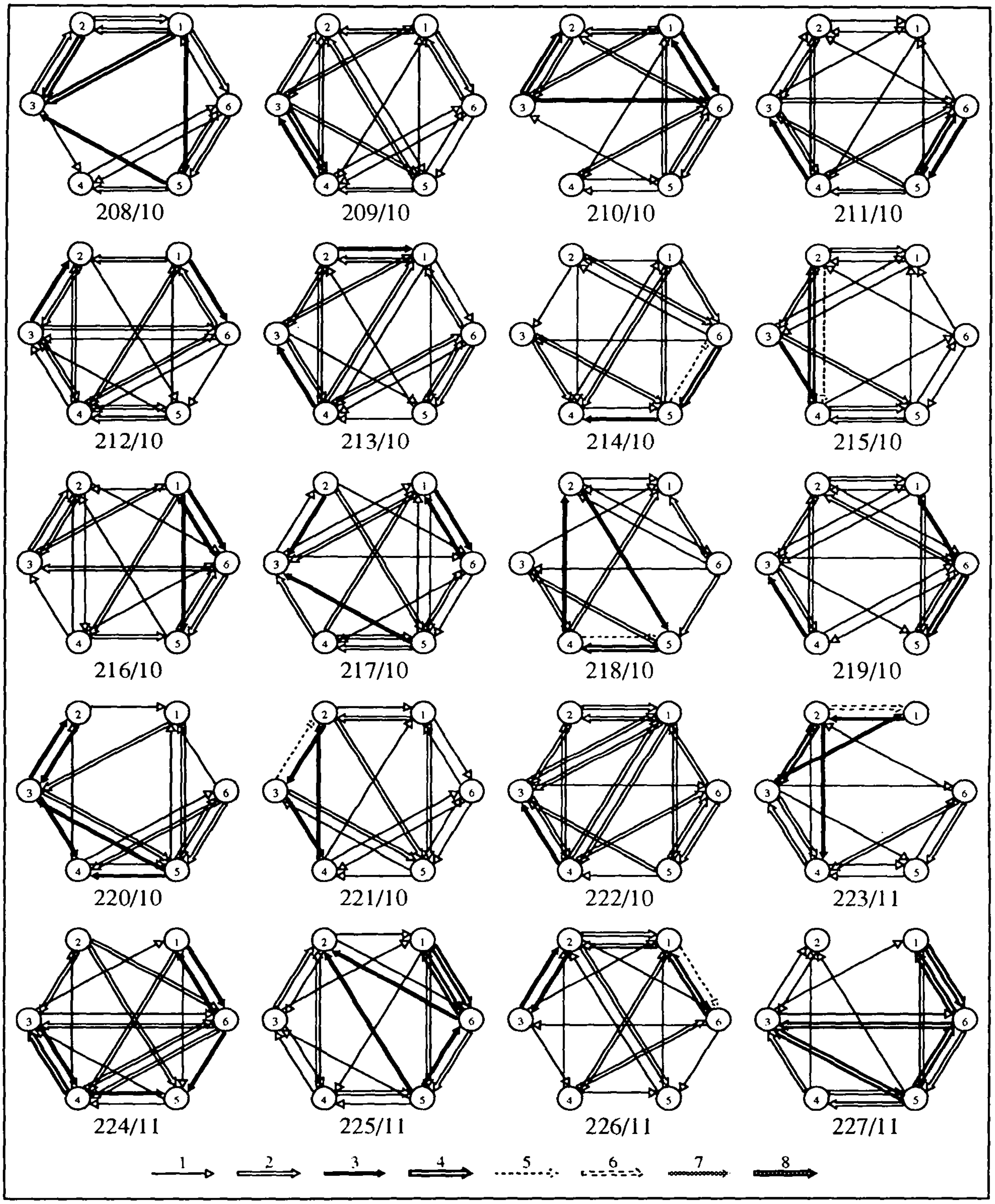
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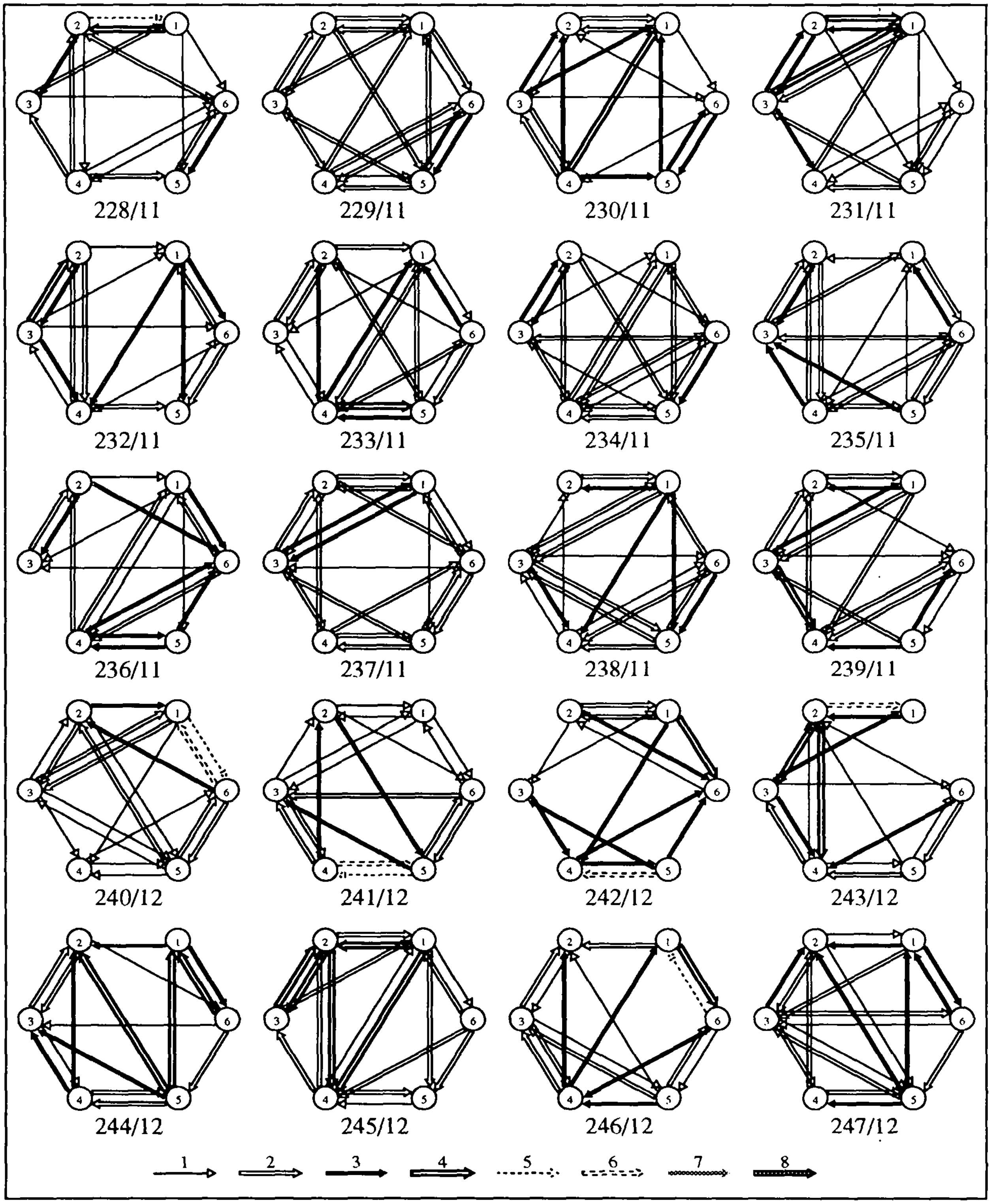
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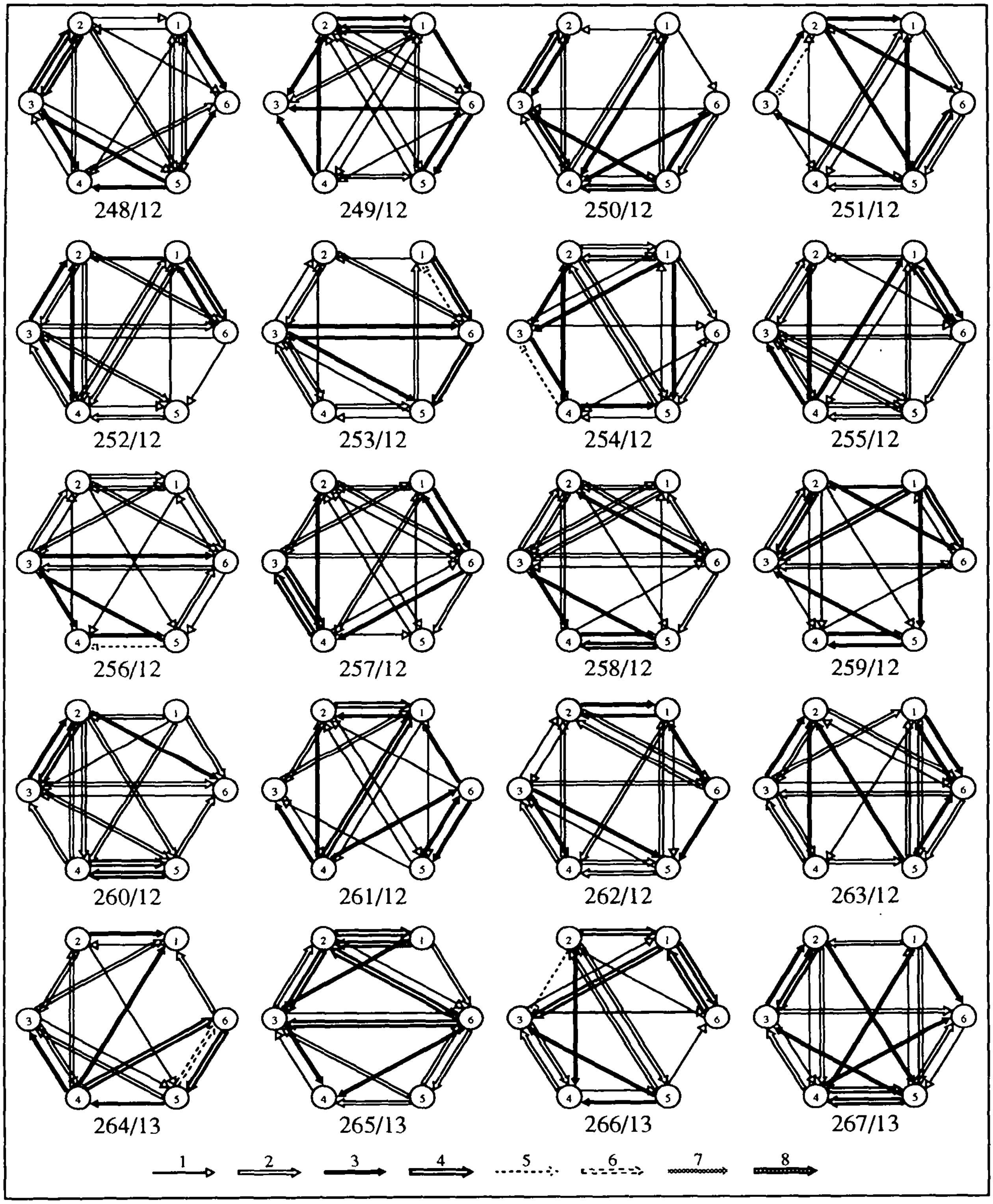
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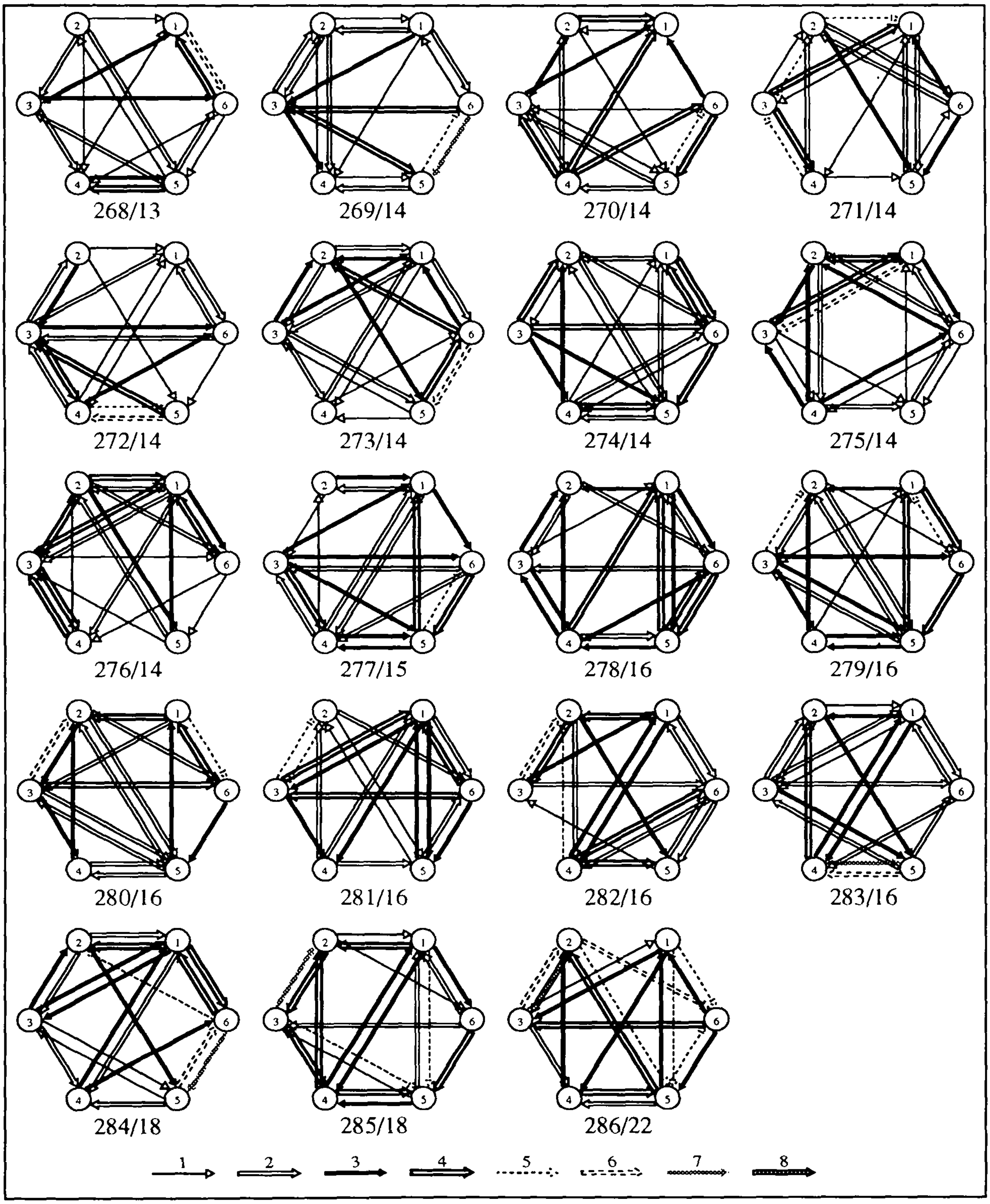
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The following table summarizes our results.

- Column 2 "Rep" indicates the number of distinct support-reduced representations obtained from one facet-defining inequality.
- Column 3 "Facets" contains the total number of distinct facet-defining inequalities belonging to a class.
- Column 4 "RHS" shows the minimum right-hand side of all the support-reduced inequalities we generated.
- Column 5 "Vertices" gives the number of vertices lying on each facet of this class.
- Column 6 indicates invariance with respect to arc-reversal, by N standing for non-invariant and Y for invariant. Note that a class is termed invariant whenever arc-reversal can also be obtained by addition of degree inequalities and permutation of nodes.
- The last column is reserved for comments on earlier work.

N°	Rep	Facets	RHS	Vertices	Inv	Comments
0	1	30	1	96	Y	non-negativity
1	2	15	1	48	Y	2 nodes subtour elimination
2	2	10	2	36	Y	3 nodes subtour elimination
3	8	120	2	42	Y	D_3^+, D_3^- [Gro77], [Fis91]
4	8	360	2	30	Y	Odd CAT (a) [Bal89]
5	8	720	3	24	N	Odd CAT (b),(c) [Bal89]
6	10	90	3	32	Y	Odd CAT (d) [Bal89]
7	8	90	3	28	Y	(c_3) [Gro77]
8	11	720	3	32	N	(c_1), (c_2) [Gro77]
9	18	180	3	32	Y	(c_4) [Gro77]
10	11	1440	3	24	N	C_3^+, C_3^- [Gro77], [Fis91]
11	11	1440	3	28	N	
12	11	1440	3	24	N	
13	14	720	4	27	Y	D_5^+, D_5^-, (d_1), (d_2) [Gro77], [Fis91]
14	15	720	4	25	Y	(d_3), (d_4) [Gro77]
15	18	360	4	30	Y	(d_5) [Gro77]
16	14	1440	4	33	N	(d_6) [Gro77]
17	8	180	4	28	Y	[Bal89], [Gro77]
18	14	1440	4	23	N	lifted subtour
19	18	720	4	30	Y	lifted subtour
20	22	1440	4	29	N	lifted subtour
21	14	180	4	28	Y	lifted subtour
22	14	720	4	25	Y	lifted subtour
23	11	720	4	26	N	lifted subtour

N°	Rep	Facets	RHS	Vertices	Inv	Comments
24	13	720	4	23	Y	lifted subtour
25	18	720	4	26	Y	lifted subtour
26	13	720	4	27	Y	lifted subtour
27	11	360	4	26	Y	[Hel53], [Kuh55]
28	14	720	4	21	Y	
29	16	360	4	30	Y	
30	14	720	4	30	Y	
31	22	720	4	26	N	
32	14	180	4	32	Y	
33	20	360	4	24	Y	
34	12	120	4	21	Y	
35	21	360	4	32	Y	
36	24	1440	5	22	N	[Hel53], [Kuh55]
37	29	1440	5	23	N	[Hel53], [Kuh55]
38	21	360	5	28	Y	[Hel53], [Kuh55]
39	27	1440	5	22	N	[Hel53], [Kuh55]
40	18	180	5	20	Y	[Hel53], [Kuh55]
41	29	1440	5	21	N	
42	27	720	5	21	Y	
43	30	1440	5	24	N	
44	24	720	5	28	Y	
45	18	720	5	26	N	
46	23	1440	5	28	N	
47	27	720	5	28	Y	
48	25	720	5	32	Y	
49	24	720	5	26	Y	
50	19	1440	5	20	N	
51	24	1440	5	25	N	
52	32	360	5	28	Y	
53	27	1440	5	19	N	
54	28	1440	5	19	N	
55	24	1440	5	19	N	
56	20	180	5	28	Y	
57	21	360	6	30	Y	[Hel53], [Kuh55]
58	18	360	6	26	Y	[Hel53], [Kuh55]
59	21	720	6	26	N	[Hel53], [Kuh55]
60	31	1440	6	22	N	
61	32	1440	6	21	N	
62	34	1440	6	21	N	
63	31	1440	6	25	N	
64	42	1440	6	26	N	

N°	Rep	Facets	RHS	Vertices	Inv	Comments
65	43	720	6	21	Y	
66	36	1440	6	21	N	
67	29	1440	6	23	N	
68	31	1440	6	22	N	
69	30	1440	6	25	N	
70	32	720	6	29	Y	
71	36	1440	6	25	N	
72	34	1440	6	23	N	
73	33	1440	6	28	N	
74	40	720	6	24	Y	
75	36	720	6	26	N	
76	36	1440	6	22	N	
77	37	1440	6	23	N	
78	26	720	6	27	Y	
79	37	720	6	28	Y	
80	38	720	6	20	Y	
81	34	1440	6	26	N	
82	27	720	6	25	Y	
83	32	360	6	29	Y	
84	37	1440	6	24	N	
85	30	720	6	26	Y	
86	34	720	6	23	Y	
87	39	120	6	26	Y	
88	37	720	6	26	Y	
89	39	1440	6	20	N	
90	36	1440	6	19	N	
91	36	720	6	21	Y	
92	42	1440	6	21	N	
93	35	720	6	25	Y	
94	28	180	6	32	Y	
95	40	720	6	20	N	
96	36	1440	7	27	N	
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98	42	1440	7	26	N	
99	36	1440	7	23	N	
100	31	1440	7	24	N	
101	45	1440	7	22	N	
102	41	1440	7	24	N	
103	41	1440	7	21	N	
104	27	1440	7	27	N	
105	37	1440	7	20	N	

N°	Rep	Facets	RHS	Vertices	Inv	Comments
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107	38	1440	7	20	N	
108	36	720	7	25	Y	
109	33	1440	7	21	N	
110	46	720	7	24	Y	
111	49	1440	7	22	N	
112	38	1440	7	25	N	
113	32	720	7	22	Y	
114	43	1440	7	21	N	
115	40	1440	7	20	N	
116	57	1440	7	20	N	
117	51	1440	7	22	N	
118	41	1440	7	21	N	
119	46	1440	7	23	N	
120	46	720	7	19	Y	
121	43	1440	7	22	N	
122	44	1440	7	19	N	
123	49	1440	7	21	N	
124	45	1440	7	19	N	
125	41	1440	7	20	N	
126	46	1440	7	20	N	
127	21	360	8	26	Y	[Hel53], [Kuh55]
128	64	1440	8	22	N	
129	58	1440	8	24	N	
130	40	1440	8	24	N	
131	57	120	8	31	Y	
132	39	720	8	30	Y	
133	42	360	8	24	Y	
134	61	1440	8	24	N	
135	53	1440	8	25	N	
136	62	720	8	24	Y	
137	55	1440	8	24	N	
138	47	1440	8	21	N	
139	49	720	8	23	Y	
140	53	1440	8	21	N	
141	56	720	8	21	Y	
142	47	1440	8	24	N	
143	54	720	8	23	Y	
144	52	1440	8	20	N	
145	48	1440	8	21	N	
146	52	1440	8	24	N	

N°	Rep	Facets	RHS	Vertices	Inv	Comments
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148	51	1440	8	20	N	
149	37	1440	8	21	N	
150	63	720	8	23	Y	
151	49	720	8	25	Y	
152	50	1440	8	22	N	
153	54	720	8	20	N	
154	40	720	8	25	Y	
155	56	1440	8	20	N	
156	57	720	8	23	Y	
157	61	720	8	23	Y	
158	52	1440	8	20	N	
159	38	1440	8	20	N	
160	61	1440	8	21	N	
161	60	1440	8	19	N	
162	53	1440	8	19	N	
163	57	1440	8	23	N	
164	60	1440	8	21	N	
165	50	720	8	26	Y	
166	36	240	8	25	Y	
167	57	720	8	23	Y	
168	62	1440	8	19	N	
169	47	720	8	23	Y	
170	46	1440	8	19	N	
171	62	1440	8	21	N	
172	52	720	8	21	Y	
173	41	1440	8	20	N	
174	40	720	8	19	Y	
175	66	1440	9	22	N	
176	56	720	9	26	Y	
177	44	1440	9	23	N	
178	44	1440	9	26	N	
179	59	1440	9	23	N	
180	61	1440	9	22	N	
181	45	1440	9	21	N	
182	47	1440	9	21	N	
183	71	720	9	22	Y	
184	57	1440	9	21	N	
185	49	1440	9	21	N	
186	60	1440	9	20	N	
187	52	1440	9	22	N	

N°	Rep	Facets	RHS	Vertices	Inv	Comments
188	60	720	9	21	Y	
189	67	1440	9	21	N	
190	61	1440	9	21	N	
191	53	1440	9	20	N	
192	55	1440	9	21	N	
193	63	1440	9	23	N	
194	82	1440	9	20	N	
195	47	1440	9	20	N	
196	53	1440	9	19	N	
197	62	1440	9	19	N	
198	61	1440	9	19	N	
199	56	360	10	32	Y	
200	52	720	10	21	Y	
201	79	1440	10	21	N	
202	63	720	10	25	Y	
203	38	720	10	21	Y	
204	73	720	10	23	Y	
205	59	1440	10	24	N	
206	74	720	10	25	Y	
207	76	1440	10	22	N	
208	62	1440	10	22	N	
209	69	1440	10	20	N	
210	62	720	10	21	Y	
211	50	1440	10	21	N	
212	68	720	10	21	Y	
213	59	720	10	21	Y	
214	72	1440	10	19	N	
215	72	1440	10	19	N	
216	79	1440	10	21	N	
217	74	1440	10	19	N	
218	73	720	10	21	Y	
219	82	720	10	21	Y	
220	82	1440	10	19	N	
221	76	1440	10	22	N	
222	64	720	10	23	Y	
223	65	1440	11	22	N	
224	74	720	11	24	Y	
225	79	1440	11	20	N	
226	65	1440	11	20	N	
227	67	1440	11	23	N	
228	61	1440	11	20	N	

N°	Rep	Facets	RHS	Vertices	Inv	Comments
229	84	1440	11	20	N	
230	75	720	11	26	Y	
231	68	720	11	23	Y	
232	58	1440	11	21	N	
233	91	1440	11	20	N	
234	85	1440	11	19	N	
235	77	1440	11	19	N	
236	63	1440	11	19	N	
237	83	1440	11	19	N	
238	88	1440	11	19	N	
239	82	1440	11	19	N	
240	93	1440	12	20	N	
241	107	1440	12	21	N	
242	53	1440	12	22	N	
243	57	1440	12	23	N	
244	79	1440	12	23	N	
245	74	720	12	25	Y	
246	87	1440	12	20	N	
247	106	1440	12	20	N	
248	85	1440	12	22	N	
249	91	720	12	23	Y	
250	74	1440	12	20	N	
251	77	1440	12	20	N	
252	79	1440	12	20	N	
253	102	1440	12	20	N	
254	75	1440	12	23	N	
255	80	1440	12	22	N	
256	85	1440	12	20	N	
257	96	720	12	21	Y	
258	95	1440	12	21	N	
259	82	1440	12	19	N	
260	76	1440	12	19	N	
261	84	1440	12	19	N	
262	109	1440	12	21	N	
263	96	720	12	20	Y	
264	82	1440	13	20	N	
265	84	1440	13	22	N	
266	89	1440	13	20	N	
267	88	720	13	22	Y	
268	69	1440	13	19	N	
269	102	720	14	21	Y	

N°	Rep	Facets	RHS	Vertices	Inv	Comments
270	89	1440	14	20	N	
271	92	1440	14	23	N	
272	99	1440	14	20	N	
273	109	1440	14	20	N	
274	102	720	14	23	Y	
275	88	1440	14	19	N	
276	112	1440	14	19	N	
277	132	1440	15	20	N	
278	119	1440	16	20	N	
279	118	1440	16	21	N	
280	85	720	16	21	Y	
281	141	1440	16	19	N	
282	119	1440	16	19	N	
283	119	1440	16	19	N	
284	152	1440	18	20	N	
285	118	1440	18	20	N	
286	149	1440	22	19	N	

Summing up the number of different facet-defining inequalities induced by every class, we obtain a total number of 319015 such inequalities describing $\boldsymbol{P}_T^6$ in a complete and irredundant manner. Also observe that column 2 indicates for each class the number of different inequalities (we generated) that define the same facet, that are also support-reduced and thus give rise to distinct facets of the monotonization of $\boldsymbol{P}_T^6$. These numbers lead to a lower bound of 17884952 different facet-defining inequalities for the monotonization of $\boldsymbol{P}_T^6$. Finally, we indicate in column 7 which inequalities are already implicitly known from the description of $\boldsymbol{P}_T^5$ obtained by [Hel53], [Kuh55].

We conclude this section with some observations:

- For $n \geq 4$, any D_{n-1}^- inequality is equivalent to a D_{n-1}^+ inequality.

  Proof: Consider a D_{n-1}^+ inequality (cf. the notation of [Fis91]) with $i_1 = 1, i_2 = 3, i_3 = 4, \ldots, i_{n-1} = n$ and node 2 as isolated node. Subtracting twice the outdegree equation for node 1, and once the corresponding equation for node n, and adding twice the indegree equation for node 2 and once the corresponding equation for node 3 yields a D_{n-1}^- inequality with $i_1 = 2, i_2 = 3, i_3 = 4, \ldots, i_{n-1} = n$ and node 1 as isolated node. ■

- For $n = 6$ and $k = 4$, the lifted subtour-inequality (c3) of [Gro77] (p. 209) defines the same facet as the odd CAT-inequality (e) of [Bal89] (p. 435).

- For $n = 6$ and $k = 5$, the total number of lifted subtour-inequalities defining distinct facets is 16, 12 more than indicated in [Gro77].

4 Conclusions and final remarks

It seems realistic to us that a complete linear description of the monotonization of P_T^6 can be obtained in a similar way. Note however that the number of vertices that are adjacent to a tour, for example, will be much higher and, thus, more computational effort will be necessary.

We conclude with some remarks on related work. By splitting up the cone induced by a given vertex and its adjacent vertices a complete description of the symmetric traveling salesman polytope on 8 nodes has been obtained in [CJR90] hereby detecting 3 new classes of facet-defining inequalities. Reinelt [Rei91] applied the same technique to the linear ordering polytope on 7 nodes. He also discovered 3 new classes of facet-defining inequalities. However, he has given no definite proof of the completeness of his system. We adapted our method to the linear ordering polytope and obtained the same results thereby confirming the completeness of Reinelt's description. Finally, we would like to mention the work of Deza, Grötschel and Laurent [DGL90] on polytopes associated with cut problems on graphs. They have introduced several classes of such polytopes and computed complete and irredundant descriptions for them over graphs of order 4 and 5.

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